Propagation of photons in resting and moving media

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The propagation of photon in a dielectric may be described with the help of the scalar and vector potentials of the medium. The main novelty of the paper is that the concept of the vector potential (which is connected with the velocity of the medium) can be extended to relativistic velocities of the medium. The position-dependent photon wave function was used to describe the propagation of the photon. The new concepts of the velocity of photon as particle and the photon mass in the dielectric medium were proposed.

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I. INTRODUCTION

Consider a photon in a dielectric medium. But what is a photon? In modern physics a photon is nothing more than quantum excitation of the electromagnetic field. We have learned from quantum electrodynamics that in the dielectric, in fact, there are no photons but polaritons, i.e., excitations of electromagnetic field coupled to the medium. However, a different point of view is also possible. Remember the case of an electron in an external electromagnetic field. On the first quantization level the electron is treated as a quantum object moving in the classical field. In this paper I develop a similar description for a photon. The photon treated as a quantum object ''feels'' the medium as an external classical field. To describe a photon in terms of a one-particle wave function, i.e., on the *first* quantization level, I follow the methods presented in Refs. [1–3]. Another approach was proposed in Ref. [4]. The concept of the position-representation photon wave function has a long history and is still controversial. Nevertheless I do not want to discuss the question here. The reader interested in this problem is referred to Refs. [1–4] and references therein.

The paper is organized in the following way. In Sec. II, I develop the description of a photon in a medium in terms of the photon wave function. Some attempts of this kind have been presented in Refs. [1–3]. What is new in my approach is to show that the influence of the medium on the photon can be described through some potentials. Generally the idea is not new, see, e.g., Refs. [5,6], but here I realize it within the formalism of the photon wave function. On this basis, in Sec. III, the nonzero mass of the photon and the concept of the velocity of the photon as a particle appear in a quite natural way. The velocity of a photon is different from the phase or the group velocity and, to my knowledge, is a new concept.

I show in Sec. IV that the motion of the dielectric can be connected with the optical analog of the vector potential. This idea has been already presented in the literature, see Refs. [7,8]. What is new here is that the concept of the vector potential of the medium can be extended for relativistic velocities of the medium. With the help of the scalar and vector potentials of the medium one can define some optical analogs of electric and magnetic fields, and the optical analog of the Lorentz force (acting on the photon in the medium). The potentials are gauge fields and the analogs of electric and magnetic fields are gauge invariant.

II. THE SCALAR POTENTIAL OF MEDIUM

In Refs. [1–3] the following form of the Schrödinger equation for free photon was proposed:

$$i\hbar\partial_t \mathsf{F} = H_f \mathsf{F}, \tag{1}$$

$$\mathsf{F} = \begin{bmatrix} \mathbf{E}(t,\mathbf{r}) + i\mathbf{H}(t,\mathbf{r}) \\ \mathbf{E}(t,\mathbf{r}) - i\mathbf{H}(t,\mathbf{r}) \end{bmatrix}, \quad H_f = c\begin{bmatrix} \mathbf{p}\cdot\mathbf{S}, 0 \\ 0, -\mathbf{p}\cdot\mathbf{S} \end{bmatrix}, \tag{2}$$

$\mathbf{p} = -i\hbar\boldsymbol{\nabla}$, momentum of photon; $(\mathbf{S}_i)_{kl} = -i\varepsilon_{ikl}$, spin photon matrix ($\varepsilon_{ikl}$, antisymmetric Levi-Civita symbol).

On the classical language, the equations are equivalent to the following Maxwell equations:

$$\partial_t\mathbf{E} = c\boldsymbol{\nabla}\times\mathbf{H}, \quad \partial_t\mathbf{H} = -c\boldsymbol{\nabla}\times\mathbf{E}, \quad \mathbf{D} = \mathbf{E}, \quad \mathbf{B} = \mathbf{H}, \tag{3}$$

describing free fields in vacuum. Since all the information carried by function F is contained in its positive energy (positive frequency) part $\mathsf{F}^{(+)}$, following Ref. [3], I take this part as the true photon wave function and denote it as ψ,

$$\psi = \mathsf{F}^{(+)}. \tag{4}$$

To become a complete set of Maxwell equations, Eq. (3) must be supplied by divergence conditions $\boldsymbol{\nabla}\cdot\mathbf{E} = 0$, $\boldsymbol{\nabla}\cdot\mathbf{H} = 0$. It is equivalent to the relation $\mathbf{p}\cdot\psi = 0$.

In order to describe the propagation of the photon in dielectric, one should include in the Hamiltonian the interaction term. On the microscopic level, such an interaction is rather complicated, but here I will take it into account in a phenomenological way. Let us begin with stationary states of the photon in a homogeneous dielectric. For a stationary state the wave function takes on the form

$$\psi_\omega = \varphi_\omega(\mathbf{r})\exp(-i\omega t), \quad \text{where} \quad \varphi_\omega(\mathbf{r}) = \begin{bmatrix} \mathbf{E}(\mathbf{r}) + i\mathbf{H}(\mathbf{r}) \\ \mathbf{E}(\mathbf{r}) - i\mathbf{H}(\mathbf{r}) \end{bmatrix}. \tag{5}$$

The propagating photon in every time and in every space point ''feels'' the same coupling with the medium. We may try to describe the interaction by a single constant coupling value Ω_ω. If some inhomogeneities are in the dielectric, then

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the interaction depends on **r** and will be modeled by a function $\Omega_\omega(\mathbf{r})$. Sometimes, I want to restrict considerations to the nonmagnetic media, i.e., $\mu=1$. It means that the medium is coupled only to the electric part of the photon wave function. Generally, the couplings of the medium with the electric and magnetic part of the wave function may be different. To take it into account I introduce two real and symmetric matrices γ and η which split the wave function ψ_ω into electric and magnetic parts

$$\gamma\psi_\omega+\eta\psi_\omega=\psi_\omega, \tag{6}$$

$$\gamma\psi_\omega=\begin{bmatrix}\mathbf{E}\\ \mathbf{E}\end{bmatrix} \quad \text{and} \quad \eta\psi_\omega=\begin{bmatrix}i\mathbf{H}\\ -i\mathbf{H}\end{bmatrix}, \tag{7}$$

$$\eta=\frac{1}{2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} \quad \text{and} \quad \gamma=\frac{1}{2}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}. \tag{8}$$

The projection operators γ, η fulfill the following relations, i.e., $\gamma^2=\gamma$, $\eta^2=\eta$, $\gamma\eta=0$. Thus the case of the propagating photon in inhomogeneous nonmagnetic dielectric can be modeled in the following way:

$$\hbar\omega\psi_\omega=H_f\psi_\omega-\Omega_\omega(\mathbf{r})\gamma\psi_\omega. \tag{9}$$

I interpret the term $\Omega_\omega(\mathbf{r})\gamma$ as a *potential energy* operator of the photon in dielectric. In order to see what is the meaning of the quantity $\Omega_\omega(\mathbf{r})$ in the classical language, one may translate Eq. (9) into the ordinary form of nonvacuum Maxwell equations

$$-i\omega[1+4\pi\chi_\omega(\mathbf{r})]\mathbf{E}=c\boldsymbol{\nabla}\times\mathbf{H}, \quad i\omega\mathbf{H}=c\boldsymbol{\nabla}\times\mathbf{E}, \tag{10}$$

$$\text{where} \quad \chi_\omega(\mathbf{r})=\frac{1}{4\pi}\frac{\Omega_\omega(\mathbf{r})}{\hbar\omega}.$$

Thus $\Omega_\omega(\mathbf{r})$ is directly connected with the dielectric susceptibility $\chi_\omega(\mathbf{r})$.

It is easy to generalize this approach and write Maxwell equations for dispersive media in the form of a Schrödinger equation. If simultaneously many frequencies are present in the medium, then it is reasonable to expect (from the quantum mechanical point of view—indistinguishable alternatives) that the interaction of medium with the photon in such a *nonstationary* state is described by superposition of single-frequency interaction terms. Thus, if the photon is described by a wave packet $\psi(t,\mathbf{r})$, then the interaction of the packet with the medium can be described by an integral operator $\hat{\Omega}_L$ in the following way:

$$\hat{\Omega}_L\psi(t,\mathbf{r})=\int\Omega_\omega(\mathbf{r})\psi_\omega(t,\mathbf{r})d\omega$$
$$=\int\Omega_\omega(\mathbf{r})\varphi_\omega(\mathbf{r})\exp(-i\omega t)d\omega. \tag{11}$$

The equation of motion of the photon takes on the quasi-Schrödinger form

$$i\hbar\partial_t\psi=H_f\psi-\gamma\hat{\Omega}_L\psi. \tag{12}$$

The integral form of the interaction term makes the relation between fields **D** and **E** nonlocal in time, i.e.,

$$\mathbf{D}(t,\mathbf{r})=\mathbf{E}(t,\mathbf{r})+4\pi\int\chi(\tau)\mathbf{E}(t-\tau,\mathbf{r})d\tau. \tag{13}$$

Equation (12) simplifies in some special cases. For example, in a *nondispersive* medium

$$\Omega_\omega(\mathbf{r})=4\pi\chi(\mathbf{r})\hbar\omega, \tag{14}$$

where $\chi(\mathbf{r})$ does not depend on ω, Eq. (12) becomes

$$i\hbar\partial_t(1+4\pi\chi(\mathbf{r})\gamma)\psi=H_f\psi. \tag{15}$$

In this case it is possible to construct some *effective* wave function (and Hamiltonian) and such an effective form is used in Refs. [1–3].

Another interesting case is when $\Omega(\mathbf{r})$ is *independent* of ω, then $\hat{\Omega}_L\psi(t,\mathbf{r})=\Omega(\mathbf{r})\psi(t,\mathbf{r})$. In this case the similarity to the case of the electron in an external field is the most appealing. For simplicity, in the next sections, I restrict the discussion to stationary states.

III. THE MASS AND VELOCITY OF PHOTON IN THE MEDIUM

When the couplings with the electrical and magnetic parts are taken into account, the Schrödinger equation takes on the form

$$\hbar\omega\psi_\omega=H_f\psi_\omega-\Omega_\omega(\mathbf{r})\gamma\psi_\omega-\Gamma_\omega(\mathbf{r})\eta\psi_\omega. \tag{16}$$

$\Omega_\omega(\mathbf{r}),\Gamma_\omega(\mathbf{r})$ have interpretation of potential energies. $\Gamma_\omega(\mathbf{r})$ is connected with magnetic susceptibility $\chi^m_\omega(\mathbf{r})=(1/4\pi)\Gamma_\omega(\mathbf{r})/\hbar\omega$. In order to obtain the connection between the *total energy* and *momentum* one may iterate this equation. In the case of a homogeneous medium one obtains

$$(\hbar\omega)^2\psi_\omega=[H_f^2-(\hbar\omega\Omega_\omega+\hbar\omega\Gamma_\omega+\Omega_\omega\Gamma_\omega)]\psi_\omega, \tag{17}$$

where the identities $H_f\gamma+\gamma H_f=H_f$, $H_f\eta+\eta H_f=H_f$ have been used.

Equation (17) is in fact the classical wave equation. It is easy to note that

$$\frac{\Omega_\omega}{\hbar\omega}+\frac{\Gamma_\omega}{\hbar\omega}+\frac{\Omega_\omega\Gamma_\omega}{(\hbar\omega)^2}=\varepsilon_\omega\mu_\omega-1=n_\omega{}^2-1, \tag{18}$$

where $\varepsilon_\omega,\mu_\omega$ are the permittivity and permeability of the medium, and n_ω is a refractive index, and also that

$$H_f{}^2=c^2(\mathbf{p}\cdot\mathbf{S})^2=c^2p^2 \quad (\mathbf{p}\cdot\psi_\omega=0). \tag{19}$$

Thus, putting $\mathbf{p}=-i\hbar\boldsymbol{\nabla}$ in Eq. (17) one obtains

$$\boldsymbol{\nabla}^2\psi_\omega+n_\omega^2\frac{\omega^2}{c^2}\psi_\omega=0. \tag{20}$$

The term $n_\omega^2\omega^2$ mixes the kinetic and potential terms of the Schrödinger equation (16). From my point of view it is more natural to interpret Eq. (17) in another way. That is, to put $E \equiv \hbar\omega$ and to rewrite Eq. (17) as a connection between E and p in the form

$$E=\sqrt{c^2p^2+\frac{(\Omega_\omega-\Gamma_\omega)^2}{4}}-\frac{1}{2}(\Omega_\omega+\Gamma_\omega). \tag{21}$$

It is apparent that E is the energy of a massive relativistic particle in an external field. Thus the photon in the dielectric gains the mass m given by

$$m^2c^4 \equiv \frac{(\Omega_\omega-\Gamma_\omega)^2}{4}. \tag{22}$$

The photon gains the mass, because of the interaction with the sea of charges in the dielectric. It reminds one of Feynman's remark: ''mass is interactions.'' The term

$$U=-\tfrac{1}{2}(\Omega_\omega+\Gamma_\omega) \tag{23}$$

is a classical potential energy. It confirms the previous interpretation of the quantities Ω_ω and Γ_ω as some potential energies. Equation (22) predicts that the mass of photon becomes zero not only in empty space (when Ω_ω and Γ_ω vanish) but also when $\Omega_\omega=\Gamma_\omega$ (equivalently $\varepsilon_\omega=\mu_\omega$).

Thus the wave equation (17) takes on the form of the Klein-Gordon equation

$$(E-U)^2\psi_\omega=(c^2p^2+m^2c^4)\psi_\omega. \tag{24}$$

Certainly, the massive photon in the dielectric has energy

$$E=\frac{mc^2}{\sqrt{1-(\nu^2/c^2)}}+U. \tag{25}$$

Putting $E\equiv\hbar\omega$ and using Eqs. (22) and (23) one can calculate from Eq. (25) the velocity ν of photon in the dielectric:

$$\frac{\nu^2}{c^2}=1-\frac{(\varepsilon_\omega-\mu_\omega)^2}{(\varepsilon_\omega+\mu_\omega)^2}, \tag{26}$$

Note that ν is the velocity of the photon *as a particle*. It never exceeds c. The remarkable feature is that ν is equal to c not only in empty space but also if $\varepsilon_\omega=\mu_\omega$ (as one should expect because then the photon mass is zero). Knowing m and ν one can calculate the photon momentum

$$p=\frac{m\nu}{\sqrt{1-(\nu^2/c^2)}}=n_\omega\frac{\hbar\omega}{c}. \tag{27}$$

The velocity ν is neither the phase nor group velocity. The phase velocity ν_{ph} of the photon is

$$\nu_{\mathrm{ph}}\equiv\frac{\omega}{k}\equiv\frac{E}{p}=\frac{c}{n_\omega}\quad(\text{where } p\equiv\hbar k). \tag{28}$$

And the group velocity ν_g is

$$\nu_g\equiv\frac{\partial\omega}{\partial k}\equiv\frac{\partial E}{\partial p}=\frac{c}{n_\omega+\omega\dfrac{\partial n}{\partial\omega}}. \tag{29}$$

In the *nondispersive* case $(\partial n/\partial\omega=0)$ the group velocity is equal to the phase velocity. On the other hand, the group velocity ν_g is equal to ν in the *independent* of ω case $(\partial\Omega/\partial\omega=0,\partial\Gamma/\partial\omega=0)$. I think that one should consider the possibility that ν is really the true velocity of the photon in the dielectric. Certainly, the velocity of photon in the dielectric medium is not the question of definition. The answer can give only an experiment.

Note that $\hbar\omega$ plays two *distinct* roles in the above description. It is the total energy and apart it is *a parameter* determining photon-medium interaction. It is the reason why the right-hand side of Eq. (21) depends on ω.

At the end of this section I briefly comment the case of the wave packet

$$\int \psi_\omega\, d\omega=\int \varphi_\omega(\mathbf{r})\exp(-i\omega t)d\omega. \tag{30}$$

For every Fourier component of the packet one may write Schrödinger equation (16) and thus the Klein-Gordon equation (24). Because of different particle velocities ν the wave packet disperses. If the dispersion of velocities is $\Delta\nu$ the width of the packet is (in one dimension) increasing in time as $\Delta x=\Delta\nu\cdot t$. The packet describes *one* photon in a nonstationary state (the energy and the mass of the photon are not precisely determined) and Δx is the region in which it is possible to detect it. Usually the beam of light contains many photons. It means that all the photons are in the same one-particle nonstationary state (30). Now, in the region Δx you can detect many photons in different points at the same time. The probability is proportional to the rate of detection, and thus to the energy density at a given point. You have a macroscopic quantum state.

IV. THE VECTOR POTENTIAL OF THE MEDIUM

Developing an analogy with the theory of charged particles it is interesting to construct and examine the consequences of the vector potential of the medium $\mathbf{A}$ in the case of the photon. Replacing $\mathbf{p}\rightarrow\mathbf{P}+\mathbf{A}$, where $\mathbf{p}$ is kinetical and $\mathbf{P}$ canonical momentum, the Schrödinger equation (16) becomes

$$\hbar\omega\psi_\omega=c(\mathbf{P}+\mathbf{A})\cdot\mathbf{S}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\psi_\omega-\Omega_\omega(\mathbf{r})\gamma\psi_\omega-\Gamma_\omega(\mathbf{r})\eta\psi_\omega. \tag{31}$$

To determine the situation, I suppose that the light source emitting photons of energies $\hbar\omega$ rests with respect to the observer. Then, as will be shown, the physical meaning of the vector potential of the medium is directly connected with the velocity $\mathbf{u}$ of the medium. Note, there is no Doppler shift between the observer and the source and therefore the observed frequency is the same as the source frequency.

Expressing the Schrödinger equation (31) in the classical language one finds

$$\frac{\omega}{c}\mathbf{D}=i\boldsymbol{\nabla}\times\mathbf{H},\quad \frac{\omega}{c}\mathbf{B}=-i\boldsymbol{\nabla}\times\mathbf{E},$$
$$\mathbf{D}=\varepsilon_{\omega}\mathbf{E}+\mathbf{a}\times\mathbf{H},\quad \mathbf{B}=\mu_{\omega}\mathbf{H}-\mathbf{a}\times\mathbf{E}, \tag{32}$$

where $\mathbf{a}=\mathbf{A}/(\hbar\omega/c)$ is a dimensionless vector potential of the medium.

This may be compared with the nonrelativistic approximation of the Minkowski relations [9,10] obliging for uniformly moving dielectric:

$$\mathbf{D}=\varepsilon_{\omega}\mathbf{E}+(\varepsilon_{\omega}\mu_{\omega}-1)\vec{\beta}\times\mathbf{H},$$
$$\mathbf{B}=\mu_{\omega}\mathbf{H}-(\varepsilon_{\omega}\mu_{\omega}-1)\vec{\beta}\times\mathbf{E}, \tag{33}$$

where $\vec{\beta}=\mathbf{u}/c$. One finds immediately the connection between vector potential $\mathbf{a}$ and the velocity $\mathbf{u}$ of the medium

$$\mathbf{a}=(\varepsilon_{\omega}\mu_{\omega}-1)\vec{\beta}. \tag{34}$$

The result is in agreement with Refs. [7],[8], where it has been obtained in another way.

If one wants to examine the purely relativistic velocities case, the form of the Schrödinger equation (31) must be changed. One reason is that in the nonrelativistic velocities case we assumed that the couplings with the medium are the same as in the case of the resting medium. It does not need to be true. The second reason is that the moving medium in fact produces anisotropy of the whole system. This is not taken into account in the nonrelativistic velocities case. Therefore one should admit that the couplings for fields perpendicular and parallel with respect to the velocity of the medium $\vec{\beta}$ may be different. Thus one should consider the following Schrödinger equation:

$$\hbar\omega\psi_{\omega}=c(\mathbf{P}+\mathbf{A})\cdot\mathbf{S}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\psi_{\omega}-\Omega_{\perp\omega}(\mathbf{r})\begin{bmatrix}\mathbf{E}_{\perp}\\ \mathbf{E}_{\perp}\end{bmatrix}$$
$$-\Omega_{\parallel\omega}(\mathbf{r})\begin{bmatrix}\mathbf{E}_{\parallel}\\ \mathbf{E}_{\parallel}\end{bmatrix}-\Gamma_{\perp\omega}(\mathbf{r})\begin{bmatrix}i\mathbf{H}_{\perp}\\ -i\mathbf{H}_{\perp}\end{bmatrix}-\Gamma_{\parallel\omega}(\mathbf{r})\begin{bmatrix}i\mathbf{H}_{\parallel}\\ -i\mathbf{H}_{\parallel}\end{bmatrix}. \tag{35}$$

Our task is to find the relativistic form of $\mathbf{A}=(\hbar\omega/c)\mathbf{a}$. I look for the solution in the form

$$\mathbf{a}=\alpha(\beta)\vec{\beta}, \tag{36}$$

where $\alpha(\beta)$ is yet an unknown function. Expressing the Schrödinger equation (35) in the classical language one obtains Maxwell equations with material relations in the form

$$\mathbf{D}=\varepsilon_{\parallel\omega}\mathbf{E}_{\parallel}+\varepsilon_{\perp\omega}\mathbf{E}_{\perp}+\alpha(\beta)\vec{\beta}\times\mathbf{H}_{\perp},$$
$$\mathbf{B}=\mu_{\parallel\omega}\mathbf{H}_{\parallel}+\mu_{\perp\omega}\mathbf{H}_{\perp}+\alpha(\beta)\vec{\beta}\times\mathbf{E}_{\perp}. \tag{37}$$

($\varepsilon_{\parallel}$ is defined as $1+\Omega_{\parallel}/\hbar\omega$, etc.). The material relations should be compared with the relativistic form of the Minkowski relations [9,10]

$$\mathbf{D}=\varepsilon_{\omega}\mathbf{E}_{\parallel}+\frac{1-\beta^2}{1-\varepsilon_{\omega}\mu_{\omega}\beta^2}\varepsilon_{\omega}\mathbf{E}_{\perp}+\frac{\varepsilon_{\omega}\mu_{\omega}-1}{1-\varepsilon_{\omega}\mu_{\omega}\beta^2}\vec{\beta}\times\mathbf{H}_{\perp},$$
$$\mathbf{B}=\mu_{\omega}\mathbf{H}_{\parallel}+\frac{1-\beta^2}{1-\varepsilon_{\omega}\mu_{\omega}\beta^2}\mu_{\omega}\mathbf{H}_{\perp}-\frac{\varepsilon_{\omega}\mu_{\omega}-1}{1-\varepsilon_{\omega}\mu_{\omega}\beta^2}\vec{\beta}\times\mathbf{E}_{\perp}. \tag{38}$$

One immediately finds

$$\alpha(\beta)=\frac{\varepsilon_{\omega}\mu_{\omega}-1}{1-\varepsilon_{\omega}\mu_{\omega}\beta^2}, \tag{39}$$

and as well the other parameters as $\varepsilon_{\parallel\omega},\varepsilon_{\perp\omega}$, etc.

For a uniformly moving homogeneous medium the result (39) is exact. To a good approximation it can be useful as well in the case of a nonuniformly moving inhomogeneous medium provided that the potentials Ω, Γ and the flow $\mathbf{u}$ vary only gradually, i.e., do not vary significantly over one optical wavelength and one optical cycle.

For nonrelativistic velocities one can immediately write the connection between energy and momentum simply by substitution in Eq. (21), $\mathbf{p}\rightarrow\mathbf{P}+\mathbf{A}$:

$$E=\sqrt{c^2(\mathbf{P}+\mathbf{A})^2+\frac{(\Omega_{\omega}-\Gamma_{\omega})^2}{4}}-\frac{1}{2}(\Omega_{\omega}+\Gamma_{\omega}). \tag{40}$$

It is possible because [as one can easily check with the help of Maxwell equations (32)] the divergence condition $\boldsymbol{\nabla}\cdot\mathbf{D}=0,\boldsymbol{\nabla}\cdot\mathbf{B}=0$ is equivalent to the condition $\mathbf{p}\cdot\psi=0$ (similarly as it was in the case of the resting medium). Equation (40) is exact for a uniformly moving medium, and a good approximation in the case of a nonuniformly moving inhomogeneous medium. Note that the mass of the photon is the same as it was in the resting medium.

If one wants to iterate the relativistic Schrödinger equation (35) it will be advantageous to write it in the more suitable form

$$\hbar\omega\psi_{\omega}=(H_f-\Omega_{\parallel\omega}R_{\parallel}\gamma-\Omega_{\perp\omega}R_{\perp}\gamma-\Gamma_{\parallel\omega}R_{\parallel}\eta$$
$$-\Gamma_{\perp\omega}R_{\perp}\omega)\psi_{\omega}, \tag{41}$$

where projection operators $R_{\parallel}$ and $R_{\perp}$ have been defined in the following way: $R_{\parallel}\mathbf{V}=\mathbf{V}_{\parallel}$ and $R_{\perp}\mathbf{V}=\mathbf{V}_{\perp}$ ($\mathbf{V}$ is a vector, $\parallel$ and $\perp$ with respect to the velocity of the medium). The $R_{\parallel}$ and $R_{\perp}$ commute with γ, η. The Hamiltonian H_f has the same structure as in the case of the resting medium but now $\mathbf{p}=\mathbf{P}+\mathbf{A}$. Other useful relations are $H_f\gamma+\gamma H_f=H_f$, $H_f\eta+\eta H_f=H_f$, and $H_fR_{\parallel}+R_{\parallel}H_f=H_f-H_f^{\parallel}$, $H_fR_{\perp}+R_{\perp}H_f=H_f+H_f^{\parallel}$. The Hamiltonian $H_f^{\parallel}$ differs from H_f in such a way that $\mathbf{p}\cdot\mathbf{S}$ is replaced by $\mathbf{p}_{\parallel}\cdot\mathbf{S}$. In general the obtained result of the iteration is rather complicated and I will not write it down. I only examine here the simplest but physically most interesting case when the light in the form of a plane wave propagates in the same direction as the medium moves. In this case the result has exactly the same form as Eq. (40) with the only difference that Ω_{ω} is replaced by $\Omega_{\perp\omega}$ and Γ_{ω} by $\Gamma_{\perp\omega}$. In particular, the mass of photon m_{β} in the relativistic case is given by

$$m_\beta c^2 \equiv \frac{|\Omega_{\perp\omega}-\Gamma_{\perp\omega}|}{2} = \frac{1}{2}\,\frac{1-\beta^2}{1-\varepsilon_\omega\mu_\omega\beta^2}|\varepsilon_\omega-\mu_\omega|\hbar\omega$$
$$= \frac{1-\beta^2}{1-\varepsilon_\omega\mu_\omega\beta^2}mc^2, \tag{42}$$

where $m=|\varepsilon_\omega-\mu_\omega|\hbar\omega/2c^2$ is the mass of the photon in the rest medium. At first, it might seem strange that the mass has changed but it must be so, because the mass of photon in the medium (never mind resting or moving) is always determined by the couplings (Ω and Γ) and these couplings in the moving medium have changed. The change of mass does not appear in the nonrelativistic velocities case because it is only the second order effect (with respect to β). In the following I restrict considerations to the nonrelativistically moving media, which, no doubt, is reasonable from a practical point of view. It is interesting that in some limited sense it is possible to develop photodynamics, describing the behavior of the photon, in close analogy to the electrodynamics, as the theory of charged particles. In particular one can determine a classical force **F** acting on the photon

$$\mathbf{F}=\dot{\mathbf{p}}=\dot{\mathbf{P}}+\dot{\mathbf{A}}. \tag{43}$$

In the case of the nonuniformly moving homogeneous medium from the Hamiltonian (40) one finds

$$\dot{\mathbf{P}}=-\frac{\partial E}{\partial \mathbf{r}}=-\frac{c^2\mathbf{p}\cdot\dfrac{\partial \mathbf{A}}{\partial \mathbf{r}}}{E-U}=-\boldsymbol{\nu}\cdot\frac{\partial \mathbf{A}}{\partial \mathbf{r}}. \tag{44}$$

Here $E-U$ and $\mathbf{p}$ are given by Eqs. (25), (27), and $\boldsymbol{\nu}$ it is the particle velocity of photon given by Eq. (26). The analogs of electric $\tilde{\mathbf{E}}$ and magnetic $\tilde{\mathbf{H}}$ fields can be defined as

$$\tilde{\mathbf{E}}=\frac{\partial \mathbf{A}}{\partial t},\quad \tilde{\mathbf{H}}=-\nabla\times\mathbf{A}. \tag{45}$$

Therefore the force **F** is nothing but the optical Lorentz force

$$\mathbf{F}=\tilde{\mathbf{E}}+\boldsymbol{\nu}\times\tilde{\mathbf{H}}. \tag{46}$$

In the more general case of a nonuniformly moving inhomogeneous medium one obtains a bit more complicated result

$$\tilde{\mathbf{E}}=\frac{\partial \mathbf{A}}{\partial t}-\frac{\partial U}{\partial \mathbf{r}}-\frac{\partial (mc^2)}{\partial \mathbf{r}}\sqrt{1-\frac{\nu^2}{c^2}}=\frac{\partial \mathbf{A}}{\partial t}$$
$$+\frac{1}{2}\left\{\left(1-\sqrt{1-\frac{\nu^2}{c^2}}\right)\frac{\partial \Omega_\omega}{\partial \mathbf{r}}+\left(1+\sqrt{1-\frac{\nu^2}{c^2}}\right)\frac{\partial \Gamma_\omega}{\partial \mathbf{r}}\right\},$$
$$\tilde{\mathbf{H}}=-\nabla\times\mathbf{A}. \tag{47}$$

Note that in the inhomogeneous medium the couplings $\Omega_\omega,\Gamma_\omega$ depend on place; therefore the mass of the photon is position dependent. The fields $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{H}}$ are gauge invariant. The change of the potentials $\mathbf{A}$, $\Omega_\omega,\Gamma_\omega$ in the following way:

$$\mathbf{A}'=\mathbf{A}+\nabla f,\quad \Omega'_\omega=\Omega_\omega-\frac{\partial f}{\partial t},\quad \Gamma'_\omega=\Gamma_\omega-\frac{\partial f}{\partial t} \tag{48}$$

does not change the fields (47). Here f is whatever function of time and space.

V. SUMMARY

I find that the influence of the medium on a photon can be described by some scalar and vector potentials. Scalar potentials are directly connected with permittivity and permeability of the medium; the vector potential is connected with the velocity of the medium. The main novelty in the paper is that the notion of vector potential of the medium can be constructed also for relativistic velocities of the medium. Additional new results are the formulas for the mass of photon in resting and moving dielectric and the velocity of the photon as a particle. The velocity is different from the phase and the group velocity. Quite interesting is the fact that the photon velocity is equal to c not only in vacuum but also if $\varepsilon_\omega=\mu_\omega$. A consequence of describing the medium through scalar and vector potentials is the existence of analogs of electric and magnetic fields, as well as the optical Lorentz force which describe the influence of the medium on the photon. Just as in the theory of charged particles, the potentials are gauge fields and the analogs of electric and magnetic fields are gauge invariant.

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